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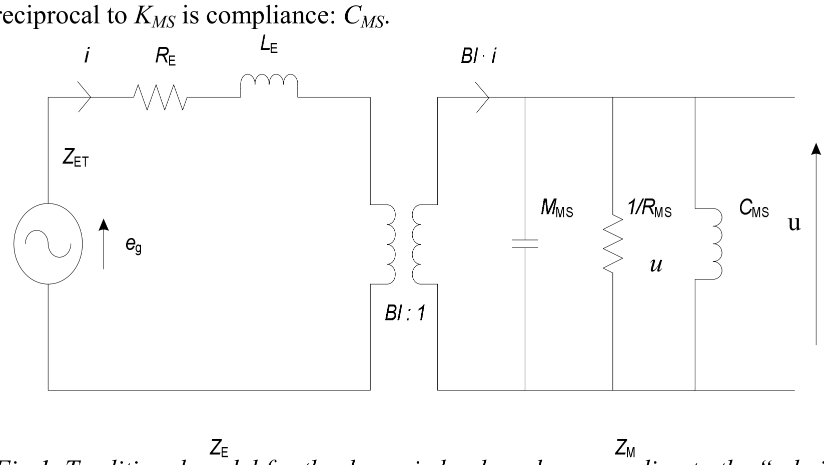
Traditional and Advanced Models for the Dynamic Loudspeaker

The traditional equivalent circuit for a loudspeaker, based on the so-called "Thiele-Small" parameters has obvious shortcomings. On the electrical side the voice coil impedance is no simple inductance, but can be represented as a shunt connection of an inductance and a "semi-inductance". Eddy current losses in a copper cap inside the air gap are adding an extra shunt resistance. On the mechanical side it is the suspension compliance and damping which show a frequency dependence not foreseen in the traditional model. In this paper is given a short survey of the traditional and the advanced models. It is followed up by second paper "Measurement of the Advanced Loudspeaker Parameters using Curve-Fitting Method".

Introduction

The motor in the electro-dynamic loudspeaker can be regarded as an electromechanical transformer.

Primary impedance is a series connection of the resistance R_E and the inductance L_E of the voice-coil. The transformer's "turn-ratio" is *Bl:1*. A primary current *i* results in a secondary "current" *Bli*; this is the force upon the mechanical system accelerating the mass M_{MS} (Newton's 2. law), being lost as heat due to viscous friction (in the mechanical resistance R_{MS}) and overcoming the restoring force due to the stiffness K_{MS} of the suspension (Hooke's law). The reciprocal to K_{MS} is compliance: C_{MS}.



Box 1

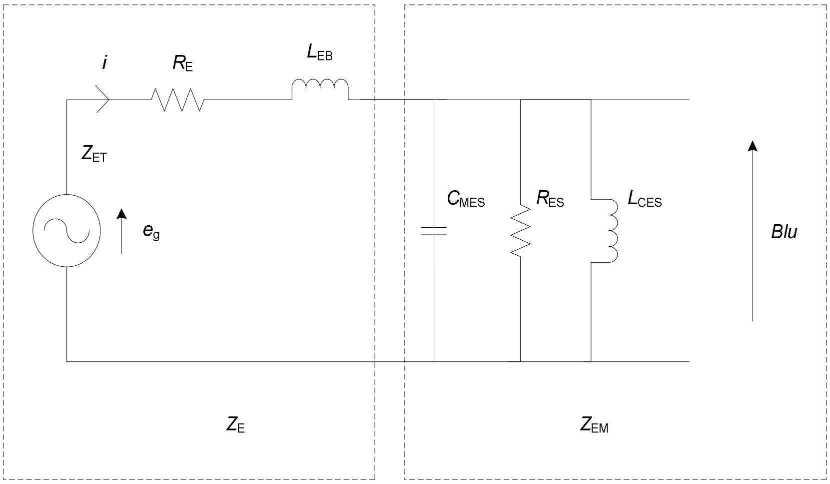
$$Bli = M_{MS} \cdot \frac{du}{dt} + R_{MS} \cdot u + \frac{1}{C_{MS}} \cdot \int u \cdot dt$$

$$i = C \cdot \frac{dV}{dt} + \frac{1}{R} \cdot V + \frac{1}{L} \cdot \int V \cdot dt$$

Fig.1. Traditional model for the dynamic loudspeaker according to the "admittance analogy"

The secondary side of the circuit shown in *Fig.1* is based on the analogy between the expressions for the force (*Bli*) and velocity (*u*) for the mechanical resonance system of the loudspeaker and for the current and voltage in an electrical parallel resonance circuit (equations shown in *Box 1*). The transformer coupling makes it possible to convert the circuit to the electrical side as shown in *Fig.2* (In *Box 2* is shown the conversion of the mechanical to the electrical parameters).

Fig.1 and *Fig 2 (below)* illustrate the traditional model for the dynamic loudspeaker according to the "admittance analogy". The circuit in the dashed left box *Fig.2* is Z_E the electrical impedance and in the right the motional impedance Z_{EM}, the electrical equivalent to Z_M in *Fig. 1*



$$L_{CES} = (Bl)^2 \cdot C_{MS} \ (H)$$

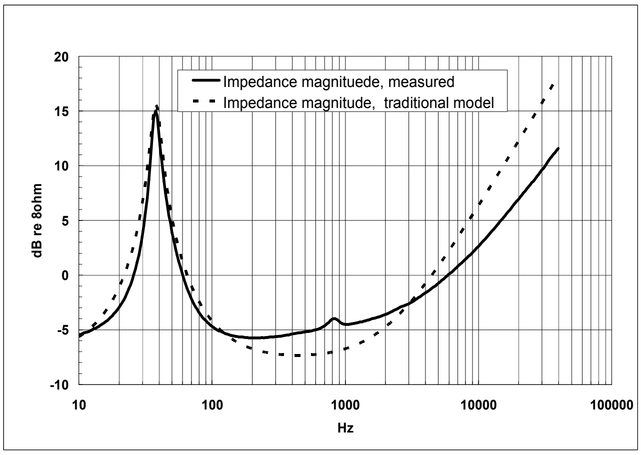
$$C_{MES} = M_{MS}/(Bl)^2 \ (F)$$

$$R_{ES} = (Bl)^2/R_{MS} \ (\Omega), \tag{2}$$

Fig.2. The model Fig.1 converted to the electrical side

1. The Need for a Better Model

The traditional model is not particularly exact. This is to be observed in *Fig.3* and *4*. In *Fig.3* is shown a simulation according to the traditional model of the loudspeaker impedance compared to a measurement and in *Fig.4* is shown the same for the electrical impedance alone (the "blocked impedance", also called so because it is measured with the voice coil in a blocked situation).



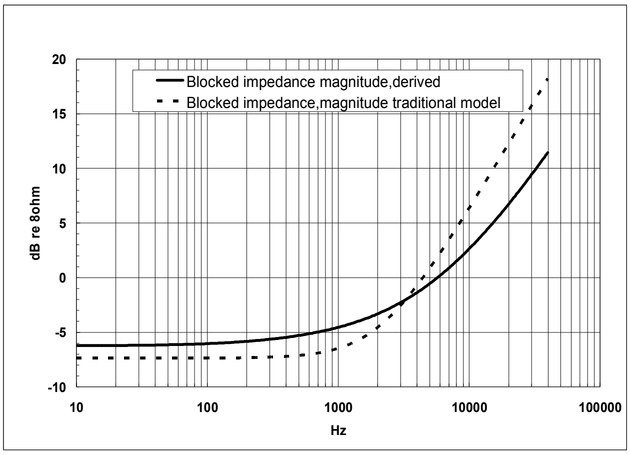


Fig. 3. Loudspeaker impedance measured and simulated according to the traditional model

Fig.4. The same for the "blocked" impedance alone

The measured curves are shown solid and the simulated dashed. The fit seems reasonable in *Fig.3* around the resonance, but *Fig.4* shows that at resonance the resistance is significantly higher than R_E. The consequence is that *Q*-values based on R_E become too low. Furthermore in a box simulation according to this model the sensitivity in the midrange becomes too high, the shape of the curve deformed and the roll-off too steep.

2. The Semi-Inductance Model for the Electrical Impedance

The main problem with the traditional model is the character of the inductance L_E not being a simple inductance, but in fact a combination of an inductance and a "semi-inductance".

The impedance of the voice-coil depends on the coil data, but also on the environment in which it is placed. The pole-piece as solid iron core for the voice-coil increases the inductance. To counteract this "ac-shorting devices" are often used; examples are shorting rings in the magnet system, aluminum spacer on top of the pole-piece and a copper cap over the pole piece.

If the solid iron pole-piece was replaced by a core of copper the result would be a transformer with the voice coil as primary and the copper core constituting a short-circuited one turn secondary. The resistance in the copper referred to primary side (multiplied by n^2 – voice coil turns squared) would then become added to the dc-resistance of the voice coil and only negligible inductance would remain.

If the solid iron instead was replaced by soft ferrite, which is electrically a non-conducting ferromagnetic material the result would be a large inductance causing early roll-off (6dB/octave).

The effect of the solid iron is something between. Iron is a conductive material and it is ferromagnetic. For such a material "skin effect" is an important factor. Both ac-current and ac-magnetic flux caused by the voice coil current are in all iron parts restricted to a "skin depth" decreasing with the square root of frequency (see the *Box 3*).

Box 3

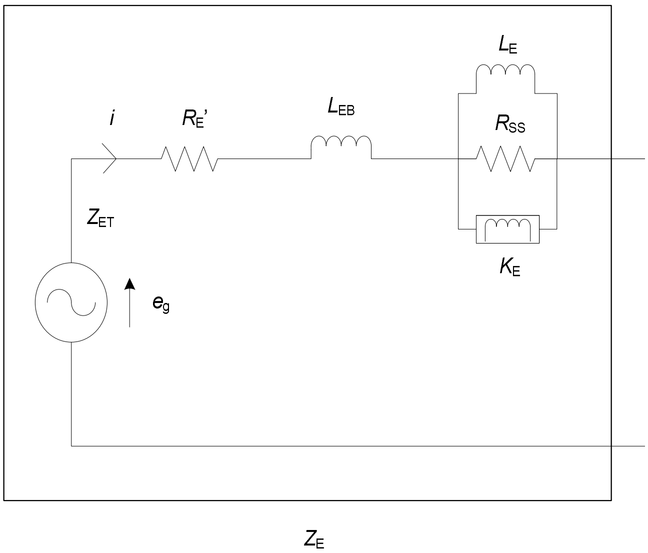
$\delta = \sqrt{\dfrac{2}{\mu\sigma\omega}}$, where δ is skin depth, μ permeability, $\omega=2\pi f$ and σ conductivity.

The resistance consequently increases with the square root of frequency and the result is a "semi-inductance"; this increases 3dB/octave instead of 6dB/octave and the phase is 45^0 instead of 90^0.

To complicate the situation the air gap is part of the magnetic circuit. For a coil with soft iron core (laminated or ferrite) and with an air gap in the magnetic circuit the inductance is calculated as the coil with the core without air gap, shunted by the coil with the reluctance of the air gap alone.

This leads for the loudspeaker voice-coil due to the solid iron core to a semi-inductance shunted by an air coil.

Outside the air gap ac-shorting devices like shorting rings and maybe an aluminum spacer on the pole-piece are placed where space is sufficient to secure "overkill" of conducting material. The consequence is that this most often will neutralize the contributions to the inductance from the "overhung" parts of the voice-coil, reducing them to minor additions in resistance to R_E then called R_E'. Inside the air gap the space is more limited and a copper cap introduces a resistance shunting the previous mentioned parallel combination of inductance and semi-inductance. This leads to the circuit shown in *Fig.5* for the electrical (blocked) impedance.



R_E' substitutes R_E and includes additions due to eddy currents in "ac-shorting devices" outside the air gap.

L_{EB} is a stray inductance due to flux around the coil out of touch with the iron - or around an upper overhung part of the voice coil in free air.

K_E is the semi-inductance, due to the solid iron core.

L_E is the inductance due to the air gap.

(In the magnetic ac-circuit a ceramic magnet acts as an extra air gap as it is electrically non-conducting and has a relative permeability close to air – as low as 1.1).

R_{SS} represents the impedance of copper (or other - non iron – conducting material) inside the air gap.

Fig.5. Semi-inductance model for the blocked impedance

3. The Motional Impedance

If the resonance frequency of a loudspeaker is lowered due to a mass added to the cone or raised due to mounting in an empty box this according to the traditional model should not change the height of the impedance peak. But so it is not in practice. The peak height increases when the resonance goes up and decreases when it goes down. This can be observed in *Fig.6*.

To cope with this a resistance proportional to frequency, $\omega\Lambda_{ES}$ is introduced in the circuit for the motional impedance as shown in *Fig.7*. It was found as an arbitrary solution giving very good fits of simulated to measured impedance curves.

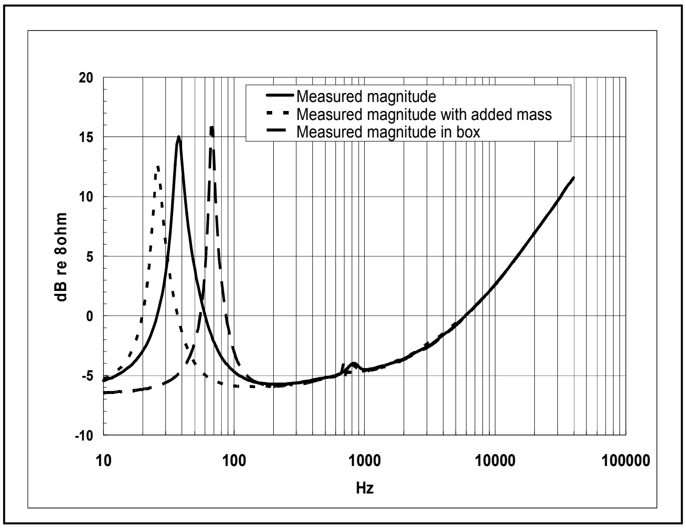


Fig.6. Loudspeaker impedance measured in free field (solid) and to the left (dashed) with mass added to the cone and to the right (dashed) mounted in an empty box

The model shown in *Fig.7* is a complete model for the loudspeaker with the semi-inductance and ac-shorting devices in- and outside the air gap and $\omega\Lambda_{ES}$ representing the frequency dependence of the damping. It is called the FDD model (frequency dependent damping).

Experience is that this model works well for the most different types of loudspeakers from dome tweeters to subwoofers, in many cases also for loudspeakers with alternative configurations of magnet structure and shorting devices.

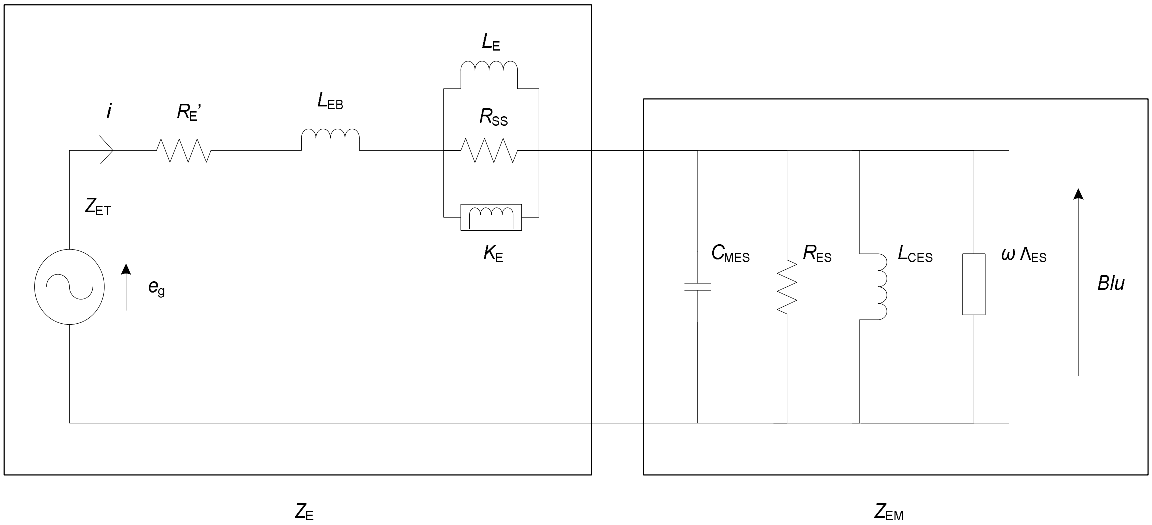


Fig. 7. The FDD/semi-induction model for the dynamic loudspeaker

Note that $\omega\Lambda_{ES}$ in this version is large compared to ωL_{CES} and shunting $L_{CES.}$ An alternative is a similar component, but small compared to ωL_{CES} in series with L_{CES}. There are good reasons for the choice made here. Λ_{ES} is robust against minor changes in L_{CES} and the addition of components representing enclosures is simpler (a closed empty box represented by a simple compliance L_B is just added as shunt to L_{CES} - not forgetting that a change in baffling might change M_{MS} a little).

The resulting resistance responsible for the Q_{MS} at resonance is found as $R_{ES}' = R_{ES} \| \omega\Lambda_{ES}$. This is a very simple function and as consequence it is reasonable to search a simple physical cause. See *Box 4*.

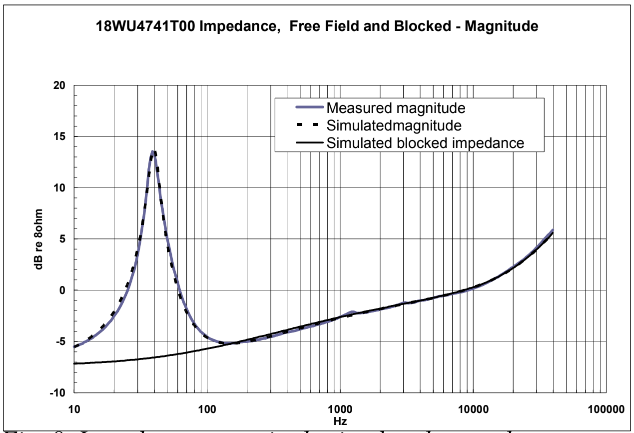


Fig. 8. Impedance, magnitude simulated according to the FDD model to the measured curve

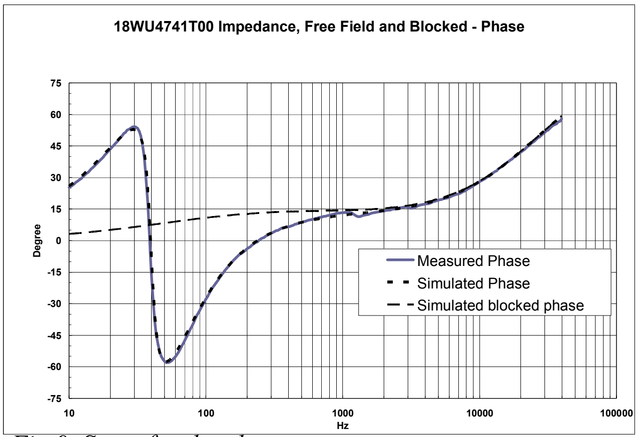


Fig.9. Same for the phase

In *Fig.8* is shown the measured impedance curve for a loudspeaker, magnitude and *Fig.9* phase. The dashed curves are the result of simulation of the corresponding curves according to the model *Fig.7.* The simulated blocked impedance is shown too (in magnitude and phase).

Box 4 Theory for frequency dependence of damping (not depending on visco-elasticity)

Going back to *Fig.1* supposing a force $Bli\ sin(\omega t)$ upon C_{MS}. This will cause amplitude:

$x = C_{MS}\ Bli\ sin(\omega t)$, but no real power is dissipated.

If the compliance of the suspension reacts with a little delay, phases lag is the result and leading to:

$x = C_{MS}\ Bli\ sin(\omega t\text{-}\varphi)$

Graphically a force/amplitude chart ($C_{MS}\ Bli\ sin(\omega t\text{-}\varphi)$ as function of $Bli\ sin(\omega t)$) becomes an ellipse (see *Fig.10)* (this is what at larger amplitudes becomes the typical hysteresis slope).

This ellipse has the area A representing the work done for one period. This area of the ellipse is found to be:

$A \approx \pi(Bli)^2\ C_{MS}\ sin\varphi = \pi\ i^2 L_{CES}\ sin\varphi$. (joule)

This is the work over one period. To get the power in watt this leads to:

$$P = \tfrac{1}{2}\ i^2\ \omega L_{CES}\ sin\varphi \quad (watt) \qquad (1)$$

In the model *Fig.7* this power is dissipated as heat in the real part of the impedance of the complex stiffness. This power is:

$$P = \tfrac{1}{2}\ i^2 \cdot Real[j\omega L_{CES} \,\|\, \omega \Lambda_{ES}] \quad = \tfrac{1}{2}\ i^2 \cdot Real[j\omega L_{CES}\ \omega\Lambda_{ES} / (j\omega L_{CES} + \omega\Lambda_{ES})]$$

$$\approx \tfrac{1}{2}\ i^2 Real[\,j\omega L_{CES}(1 - jL_{CES}/\Lambda_{ES})] \qquad (for\ L_{CES} << \Lambda_{ES})$$

$$P \approx \tfrac{1}{2}\ i^2 \omega L_{CES}^2 / \Lambda_{ES} \qquad (2)$$

From *(1)* and *(2)* is found for the value of $cos\varphi$:

$$sin\varphi \approx L_{CES} / \Lambda_{ES} \qquad (3)$$

In the FDD model the frequency dependence of the compliance is neglected and Λ_{ES} is found by curve-fitting and includes the impact of visco-elasticity of the suspension.

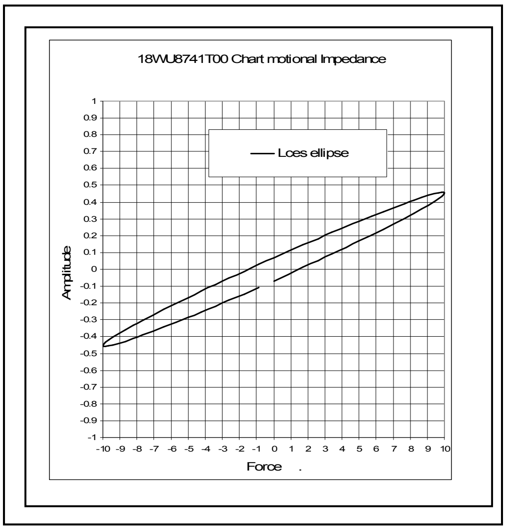


Fig. 10 Amplitude force relation for the example speaker

4. Discussion and Conclusion

Also the compliance has a certain degree of frequency dependence. This is due to a property called visco-elasticity more or less present in the suspension material. This is also the cause to creep. When a loudspeaker is connected to a dc-source the cone moves fast to the expected amplitude, but then creeps a little further. This phenomenon has no special importance for a loudspeaker's performance at audio frequencies, but it is accompanied by frequency dependence of the compliance and is also contributing with frequency dependent loss (in the FDD model included in Λ_{ES}). For surrounds with high visco-elasticity this plays the dominating role. Due to visco-elasticity compliance increases a little toward lower frequencies, so the resonance with added mass becomes lower and the resonance in a box becomes higher than expected, but as the air stiffness added by the box most often is higher than the stiffness of the speaker itself it is without real significance.

In the present paper the FDD/Semi-Inductance model for the dynamic loudspeaker is presented as an alternative to the traditional model. This new model is explained as a physical model and a theory is proposed to explain the frequency dependence of damping - independent of earlier theories based on visco-elasticity. In a second paper it will be shown how the advanced parameters can be derived using curve fitting technique based on very precise simulations according to the new model of the loudspeakers impedance in magnitude and phase.

Literature

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